

What is a circle?

In a picture:



Def: A circle is the set of points in the plane that are a fixed distance from a given point.

- We call the given point the center and the fixed distance the radius of the circle.

- any diameter (line through the center) of a circle is the same length

We can ask what the distance around the outside of the circle is and what the area inside the circle is.

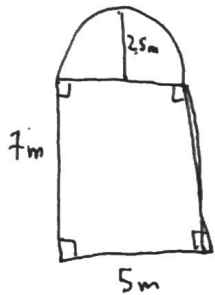
Circumference

- the circumference of a circle is the distance around the outside

$$\text{We define } \pi = \frac{C}{d} = \frac{C}{2r}. \quad \pi \approx 3.14 \text{ or } \frac{22}{7}.$$

So for any circle, $C = 2\pi r = \pi d$.

ex: Find the perimeter of the window below



3 sides of rectangle: $7+7+5 = 19 \text{ m}$

perimeter of semi-circle (half circle):

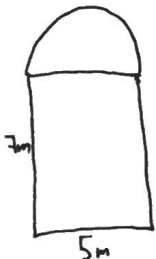
$$= \frac{1}{2} \text{ perimeter of circle} = \frac{1}{2} (2\pi \cdot 2.5) = 2.5\pi \approx 7.85 \text{ m}$$

$$\text{So the total perimeter is } 19 + 7.85 = \boxed{26.85 \text{ m}}$$

Area

- The area of a circle of radius r is $A = \pi r^2$

ex: Find the area of the window above.

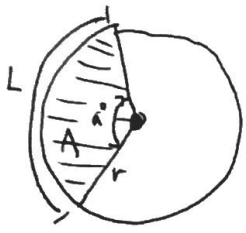


$$\text{Area of } \square: 7 \cdot 5 = 35 \text{ m}$$

$$\text{Area of } \frown: \frac{1}{2} \text{ area of full circle} = \frac{1}{2} \cdot \pi \cdot 5^2 = 12.5\pi \text{ m} \approx 39.25 \text{ m}$$

$$\text{Total area: } 35 + 39.25 = \boxed{74.25 \text{ m}}$$

Sectors of Circles



A sector is the region of a circle between two radii of the circle.

The key idea for finding the ^(A) area of a sector or ^(L) arc length of a sector or central angle (a) of a sector is that these ratios to the appropriate quantity of the full circle are equal:

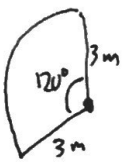
$$\text{e.g. } \frac{a^\circ}{360^\circ} = \frac{A}{\pi r^2} = \frac{L}{2\pi r}$$

Usually, we know the measure of the angle a° and find A or L :

$$A = \left(\frac{a^\circ}{360^\circ}\right) \pi r^2, \quad L = \left(\frac{a^\circ}{360^\circ}\right) \cdot 2\pi r$$

ex: We used this before for semi-circles: $a = 180^\circ \Rightarrow \frac{a^\circ}{360^\circ} = \frac{1}{2}$

Find the area of this sector:



$$A = \left(\frac{120^\circ}{360^\circ}\right) \cdot \pi \cdot 3^2 = \boxed{3\pi \text{ m}^2}$$

$\hat{=}$ this sector is $\frac{1}{3}$ of a full circle.